PMM U.S.S.R.,Vol.44,pp.421-424 0021-8928/81/4 0421 $7.50/0

UDC 62-50

ON A GUIDANCE GAME PROBLEM WITH INCOMPLETE INFORMATION*

V. D. BATUKHTIN

The formalization of a guidance problem with incomplete information on the phase state of the system being tracked is described. Conditions for the solvability of this problem are obtained. Among the basic elements of the construction described are the actually realized motions of the pursuer and the pursued. The solution's structure is determined by analogy with the extremal construction from /1/ in the form of aiming at a stable bridge. The article is closely allied with the studies in /1—8/.

1. Let a conflict controlled system consist of two objects: the pursuer and the pursued, whose motions are described by the differential equations

$$y^{\cdot} = f^{(1)}(t, y, u), \quad z^{\cdot} = f^{(2)}(t, z, v) \quad u \in P, \quad v \in Q \tag{1.1}$$

Here y and z are the n-dimensional phase vectors of, respectively, the pursuer (first player) and the pursued (second player); u and v are their controls subject to the constraints indicated (P and Q are compacta in appropriate finite-dimensional spaces); $f^{(1)}(t, y, u)$ and $f^{(2)}(t, z, v)$ are functions continuous in all arguments, satisfying a Lipschitz condition in y and z, respectively. Also, the condition of uniform continuability of the solutions of (1.1) onto any finite time interval $[t_0, \vartheta_0]$ is assumed fulfilled.

The first player has the capability of measuring at each current instant t the phase vector $y(t)$ and the signal $z^*(t)$ satisfying the constraint

$$\| z(t) - z^*(t) \| \leqslant \beta \tag{1.2}$$

The symbol $\|q\|$ denotes the norm of vector q in an appropriate finite-dimensional space, $z(t)$ is the phase vector z actually realized at instant t, β is an arbitrary constant. Thus, the actual phase vector $z(t)$ lies in a ball of radius β centered at point $z^*(t)$. The first player knows also the domain $G_0 = G(t_0)$ of possible initial states $z_0 = z(t_0)$ of the second player. When forming his own control at each instant t the player being pursued can use information on the phase vectors $y(t)$ and $z(t)$. The finite interval $[t_0, \vartheta_0]$ and the pursuer's domain of influence $L(y)$—a bounded closed set - are specified. The first player's purpose is to capture the second player within his influence domain by the instant ϑ_0; the second player's purpose is to avoid capture.

2. In conformity with the above-presented contensive description of the game problem it is assumed that the first player has the capability of continuous observation, so that at each instant he knowns the collection (t, y, z^*), where z^* is the signal. Receiving this information, the first player can construct for each t an information set, i.e., the domain of possible phase states of the second player. This set is determined constructively by a passage to the limit in a suitable approximation scheme. By $Z(\cdot, [t_0, t])$ we denote the set of all program motions $z(\cdot) = z(\cdot, t_0, z_0, \nu_{(\cdot)}(dv))$ of the second player on $[t_0, t]$, defined by the equality

$$z(t) = z_0 + \int\limits_{[t_0, t]} \int\limits_Q f^{(2)}(\tau, z(\tau), v)\, \nu_\tau(dv)\, d\tau \tag{2.1}$$

where $z_0 \in G_0$ and $\nu_\tau(dv)$ are functions weakly measurable in τ and having as their values for each $\tau \in [t_0, \vartheta_0]$ probability measures on Q. By $G(t_0, z_0, \vartheta)$ we denote the second player's attainability domain /1/ from the initial state (t_0, z_0) by the instant $t = \vartheta$ $(t_0 \leqslant \vartheta \leqslant \vartheta_0)$ as he scans all possible program controls $\nu_{(\cdot)}(dv) \in \{\nu_{(\cdot)}(dv)\}$. Suppose that a system $\Delta^{(k)}$ of non-intersecting half-open intervals $(\tau_i^{(k)}, \tau_{i+1}^{(k)}]$ $(k = 1, 2, \ldots;$ $i = 0, 1, \ldots, m(k),$ $\tau_0^{(k)} = t_0,$ $\tau_{m(k)}^{(k)} = \vartheta)$ has been chosen, covering the half-open interval $(t_0, \vartheta]$, such that $\lim \sup_i [\tau_{i+1}^{(k)} - \tau_i^{(k)}] \to 0$ as $k \to \infty$. By the instant $\vartheta \in [t_0, \vartheta_0]$ let there be realized the program motion $z(\cdot) = z(\cdot, t_0, z_0, \nu_{(\cdot)}(dv))$, $z_0 \in G_0$ and the function $z^*(\cdot) = z^*(\cdot, z(\cdot))$ whose values at each $t \in [t_0, \vartheta]$ form the signal $z^* = z^*(t)$, satisfying constraint (1.2) for the specified motion $z(\cdot)$. We say that such functions $z^*(\cdot) = z^*(\cdot, z(\cdot))$ are admissible realization of the signal for the given motion $z(\cdot)$. We define an approximate information set $G_{\Delta(k)}(\vartheta, z^*(\cdot))$ for a fixed admissible realization $z^*(\cdot)$ by the relation

$$G_{\Delta(k)}(\vartheta, z^*(\cdot)) = \{z = z(\vartheta) : \exists z(\cdot) \in Z(\cdot, [t_0, \vartheta]), \quad \forall \tau_i^{(k)} \in [t_0, \vartheta], \quad \| z(\tau_i^{(k)}) - z^*(\tau_i^{(k)}) \| \leqslant \beta\} \tag{2.2}$$

*Prikl.Matem.Mekhan.,44,No.4,595-601,1980

We remark that the set $G_{\Delta(k)}(\vartheta, z^*(\cdot))$ can be constructed for any instant $\vartheta \in [t_0\, \vartheta_0]$ if we know only the set

$$H(\tau^{(k)}_{m(k)-1}, z^*(\tau^{(k)}_{m(k)-1})) = G_{\Delta(k)}(\tau^{(k)}_{m(k)-1}, z^*(\cdot)) \cap S(z^*(\tau^{(k)}_{m(k)-1}))$$

as the admissibility domain

$$G_{\Delta(k)}(\vartheta, z^*(\cdot)) = G(\tau^{(k)}_{m(k)-1}, H(\tau^{(k)}_{m(k)-1}, z^*(\tau^{(k)}_{m(k)-1}), \vartheta)) = \bigcup G(\tau^{(k)}_{m(k)-1}, z, \vartheta)$$

Here $S(z^*(\tau^{(k)}_{m(k)-1}))$ is a ball of radius β with center at point $z^*(\tau^{(k)}_{m(k)-1})$, $G_{\Delta(k)}(\tau^{(k)}_{m(k)-1}, z^*(\cdot))$ is the approximate information set for the instant $\tau^{(k)}_{m(k)-1}$, and the union is taken over all $z \in H(\tau^{(k)}_{m(k)-1}, z^*(\tau^{(k)}_{m(k)-1}), \vartheta)$. If, however, we know only G_0 and $z^*(\cdot)$, then the set $G_{\Delta(k)}(\vartheta, z^*(\cdot))$ can be constructed recurrently.

We now define the sets $G_{\Delta(k)}(t, z^*(\cdot))$, $t \in (\tau^{(k)}_i, \tau^{(k)}_{i+1}]$, for each fixed $i = 0, 1, \ldots, m(k)$; $k = 1, 2, \ldots$ and for a chosen admissible realization $z^*(\cdot)$ by the relations

$$G_{\Delta(k)}(t, z^*(\cdot)) = \{z: z \in G(\tau^{(k)}_i, H(\tau^{(k)}_i, z^*(\cdot)), t)\}, \quad t \in (\tau^{(k)}_i, \tau^{(k)}_{i+1}], \quad G_{\Delta(k)}(t_0, z^*(\cdot)) = G_0 \tag{2.3}$$

The collection $G_{\Delta(k)}(\cdot, z^*(\cdot))$ of sets $G_{\Delta(k)}(t, z^*(\cdot))$ as t ranges the whole interval $[t_0, \vartheta_0]$ is called the approximate information motion corresponding to the specified admissible realization $z^*(\cdot)$. The section of motion $G_{\Delta(k)}(\cdot, z^*(\cdot))$ by the hyperplane $t = \tau^{(k)}_i$, $i = 0, 1, \ldots, m(k)$, is the approximate information set $G_{\Delta(k)}(\tau^{(k)}_i, z^*(\cdot))$. We can construct the sets $G_{\Delta(k)}(\cdot, z^*(\cdot))$ corresponding to all possible admissible realizations $z^*(\cdot) = z^*(\cdot, z(\cdot))$ on $[t_0, \vartheta_0]$ for each program motion $z(\cdot) \in Z(\cdot, [t_0, \vartheta_0]$. Each of the sets $G_{\Delta(k)}(\cdot, z^*(\cdot))$ is a compactum in space $[t_0, \vartheta_0] \times R^n$.

Let set G_0 be a compactum from the set $\{K_{Z_0}\}$, $K_{Z_0} \in \operatorname{comp}(R^n)$. Then the set of all collections $(t, z = z(t))$ such that at least one motion $z(\cdot) \in Z(\cdot, [t_0, \vartheta_0])$ passes through each of the points $z = z(t)$, belongs to some compactum $[t_0, \vartheta_0] \times K_Z$. This compactum can be metrized in standard fashion. For definiteness we introduce the metric

$$r = \max(|t_2 - t_1|, \|z(t_1) - z(t_2)\|), \quad t_1, t_2 \in [t_0, \vartheta_0] \tag{2.4}$$

on compactum $[t_0, \vartheta_0] \times K_Z$. By $\{G_{\Delta(k)}\}$ we denote the set of all compacta $G_{\Delta(k)} \subset [t_0, \vartheta_0] \times K_Z$ each of which coincides with some set $G_{\Delta(k)}(t, z^*(\cdot))$ of (2.3), $k = 1, 2, \ldots$, when t ranges the interval $[t_0, \vartheta_0]$. From any sequence $G_{\Delta(k)}$ $(k = 1, 2, \ldots)$ we can pick out /9/ a Hausdorff-convergent subsequence $G_{\Delta(k_j)}$ $(j = 1, 2, \ldots)$ in the sense of distance (2.4). Every compactum $G \subset [t_0, \vartheta_0] \times K_Z$ which is the Hausdorff limit for a suitable sequence $G_{\Delta(k)}$ is called on information motion $G(\cdot)$. The section $G(\vartheta)$ of information motion $G(\cdot)$ by the hyperplane $t = \vartheta$ is called the information set for each $\vartheta \in [t_0, \vartheta_0]$. The following lemma is valid.

Lemma 2.1. The information sets $G(t)$ are nonempty for each $t \in [t_0, \vartheta_0]$.

We remark that for a fixed function $z^*(t)$ $(t \in [t_0, \vartheta_0])$ and for a specified G_0 we can, in general, construct for each t a nonunique information set $G(t)$ in correspondence with the set of all possible coverings of the half-open interval $(t_0, \vartheta_0]$ and, consequently, with a nonunique Hausdorff limit G for the various sequences $G_{\Delta(k)}$.

Note. For a fixed admissible function $z^*(\cdot) = z^*(\cdot, z(\cdot))$ we define a set $\Omega(t, z^*(\cdot))$ by the relation

$$\Omega(t, z^*(\cdot)) = \{z: \exists z(\cdot) \in Z(\cdot, [t_0, t]), \quad \forall \tau \in [t_0, t], \|z(\tau) - z^*(\tau)\| \leqslant \beta\}$$

Let function $z^*(\cdot)$ be left-continuous. Then $\Omega(t, z^*(\cdot)) = G(t, z^*(\cdot))$, where $G(t, z^*(\cdot))$ is the section by hyperplane $\tau = t$ of information motion $G(\cdot, z^*(\cdot))$ obtained by the above-described passage to the limit with the specified $z^*(\cdot)$. In the more general case being considered the set $G(t, z^*(\cdot))$ is, generally speaking, wider than $\Omega(t, z^*(\cdot))$. The information set $\Omega(t, z^*(\cdot))$ is one of the main elements in minimax filtering problems /6/.

3. Let us formalize the encounter game problem described in Sect.1. Starting from the first player's information capabilities we define a strategy U as every mapping from (t, y, G) into P, which associated with each collection (t, y, G), where $G = G(t)$ is the information set, a point $u \in P$. Following /1/, every mapping from (t, y, z) into Q, that associates a point $v \in Q$ with each position (t, y, z) of system (1.1), is called the second player's strategy $V \div v(t, y, z)$. We define an admissible method of forming the signal $z^*[\tau] = z^*(\tau, z[\tau])$ for any motion $z[\tau] = z[\tau, t, z, V]$, $z \in G$, $\tau \geqslant t$, generated by strategy V as any method of forming it such that constraint (1.2) with the given motion $z[\tau] \in \{z[\tau, t, z, V]\}$ is satisfied by signal $z^*[\tau]$ for each $\tau \in [t, \vartheta_0]$ and such that the physical realizability condition is fulfilled /11/.

Suppose that an initial collection (t_*, y_*, G_*) has been given and a strategy $U \div u(t, y, G)$ has been chosen. Further, suppose that $z[t]$ is some motion of the second player, generated by the realization $\nu_t[dv]$ of his control, and that $z^*[t] = z^*(t, z[t])$ is an admissible realization of signal z^* for this motion $z[t]$, generated by some admissible method of forming the signal. Using the above-described limit passage we can construct the information motion $G(\cdot, z^*[\cdot])$ for the specified G_* and $z^*[\cdot]$. We define the Euler polygonal line $y_{\Delta(k)}[t] = y_{\Delta(k)}[t, t_*, y_*, G_*, U, z^*(\cdot, z[\cdot])]$ as an absolutely continuous vector-valued function satisfying for almost all $t \in [t_*, \vartheta_0]$ the equation

$$\frac{dy_{\Delta(k)}[t]}{dt} = f^{(1)}(t, y_{\Delta(k)}[t], u[\tau_i^{(k)}]), \quad \tau_i^{(k)} \leqslant t < \tau_{i+1}^{(k)}, \quad i = 0, 1, \ldots, m(k) \tag{3.1}$$

$$u[\tau_i^{(k)}] = u(\tau_i^{(k)}, y_{\Delta(k)}[\tau_i^{(k)}], G[\tau_i^{(k)}]), \qquad G[\tau_i^{(k)}] = G(\tau_i^{(k)}, z^*(\cdot, z[\cdot])$$

where $G[\tau_i^{(k)}]$ is the information set for instant $\tau_i^{(k)}$, being, consequently, a section of information motion $G(\cdot, z^*[\cdot])$ by hyperplane $t = \tau_i^{(k)}$. Thus, the realization of the Euler polygonal line $y_{\Delta(k)}[t]$ for a fixed collection (t_*, y_*, G_*) depends, for a selected strategy U, on the realization of motion $z[\cdot]$ and on the realization, generated by some admissible forming method, of the signal $z^*(\cdot, z[\cdot])$ for this motion $z[\cdot]$. The set $Y_{\Delta(k)}[t, t_*, y_*, G_*, U]$ of all Euler polygonal lines generated from (t_*, y_*, G_*) by strategy U for a fixed covering $\Delta^{(k)}$ can be constructed by the rule

$$Y_{\Delta(k)}[t, t_*, y_*, G_*, U] = \bigcup (\bigcup y_{\Delta(k)}[t, t_*, y_*, G_*, U, \quad z^*(\cdot, z[\cdot])]), \ z[\cdot] \ z^*(\cdot, z[\cdot])$$

The union within the parentheses is taken over all admissible realizations of signal $z^*(\cdot, z[\cdot])$ for a fixed motion $z[\cdot]$; the union outside the parentheses is taken over all realizations of motion $z[\cdot]$ which are identified with all program motions $z(\cdot)$ of the second player.

Let us consider the sequence of pairs

$$\{y_{\Delta(k)}[\cdot], \ G(\cdot, z^*(\cdot, z^{(k)}[\cdot]))\}, \ y_{\Delta(k)}[\cdot] = y_{\Delta(k)}[\cdot, t_*, y_*^{(k)}, \quad G_*^{(k)}, \ U, \ z^*(\cdot, z^{(k)}[\cdot])]$$

From this sequence we can pick out a convergent subsequence $\{y_{\Delta(kj)}[\cdot], G(\cdot, z^*(\cdot, z^{(kj)}[\cdot]))\}$ in which the subsequence $y_{\Delta(kj)}[\cdot]$ converges uniformly on $[t_*, \vartheta_0]$ to $y[t]$ as $j \to \infty$, while the subsequence $G(\cdot, z^*(\cdot, z^{(kj)}[\cdot]))$ Hausdorff-converges in the sense of metric (2.4) to the compactum $G[\cdot] \subset [t_*, \vartheta_0] \times K_Z$. Every function $y[t]$ for which we can find, on every interval $[t_*, \vartheta_0]$, a sequence $\{y_{\Delta(k)}[\cdot], \ G(\cdot, z^*(\cdot, z^{(k)}[\cdot]))\}$ converging in the above-mentioned sense to $\{y[\cdot], G[\cdot]\}$ as $k \to \infty$, is called a motion $y[t] = y[t, t_*, y_*, G_*, U]$ generated by a strategy $U \div u(t, y, G)$ from the initial collection (t_*, y_*, G_*). We can now state the encounter problem.

Problem. Given an initial collection (t_0, y_0, G_0). Find the strategy $U \div u(t, y, G)$ ensuring the contact

$$z[t] \in L(y[t]), \quad t \in [t_0, \vartheta_0]$$

for every motion $y[t] = y[t, t_0, y_0, G_0, U]$.

4. Let us set up the extremal construction as applicable to the approach problem posed. By $W(t)$ we denote the set of sections by hyperplane $\tau = t$, $\tau \in [t_0, \vartheta_0]$, i.e., of components of collections $(y(\tau), \ G(\tau))$ of all program motions $y(\tau) = y(\tau, t_*, y_*, \mu_\tau(du))$, $t_* \in [t_0, \vartheta_0]$, $y_* \in K_{Y_*}$, $\mu_\tau(du) \in \{\mu_\tau(du)\}$ ($\mu_\tau(du)$ is the first player's program control), and of all information motions $G(\tau) = G(\tau, z^*(\tau, z(\tau)))$ generated by all possible program motions $z(\tau) \in Z(\cdot, [t_*, t])$, $\tau \in [t_*, t]$, and by the corresponding admissible realizations of signal $z^*(\tau, z(\tau))$ when scanning all inital conditions $(z_*, G_*) \in K_{Z_*} \times \{K_{Z_*}\}$. When $t_* = t_0$ we have $z_* = z_0$, $G_* = G_0$, $(z_0, G_0) \in K_{Z_0} \times \{K_{Z_0}\}$. Set $W(t)$ is nonempty and closed for each $t \in [t_0, \vartheta_0]$. We say that the system of sets $W(t)$ $(t_0 \leqslant t \leqslant \vartheta_0)$ is u-stable relative to set $M = \{(y, G)\colon G \subset L(y)\}$ if for any $t_* \in [t_0, \vartheta_0]$, any collection $(y_*, G_*) \in W(t_*)$, any program motion $z(\cdot)$, any admissible realization of signal $z^*(\cdot) = z^*(\cdot, z(\cdot))$, any information motion $G^*(\cdot, z^*(\cdot))$, and any $\delta \in (0, \vartheta_0 - t_*)$ we can find a motion $y^*(\cdot) = y(\cdot, t_*, y_*, \mu_{(\cdot)}^*(du))$ of the first player such that either the inclusion $(y^*(t_* + \delta), \ G^*(t_* + \delta, z^*(\cdot))) \in W(t_* + \delta)$ is ensured or $(y^*(t), G^*(t, z^*(\cdot))) \subset M$ for some $t \in [t_*, t_* + \delta]$.

We introduce the notion of the first player's strategy extremal to $W(t)$. Suppose that the collection $(t, y[t], \ G[t])$ is realized at instant $t \in [t_0, \vartheta_0]$. By $(y_W{}^\circ[t], \ G_W{}^\circ[t])$ we denote a collection from $W(t)$ closest to $(y[t], G[t])$ in the sense of the distance

$$\rho[t] = \max(\| y[t] - y_W{}^\circ[t] \|, \ d(G[t], \ G_W{}^\circ[t])) \tag{4.1}$$

$$d(G[t], \ G_W{}^\circ[t]) = \max_{z \in G[t]} d(z, G_W{}^\circ[t]) \tag{4.2}$$

where $d(z, G_W{}^\circ[t])$ is the distance from point z to set $G_W{}^\circ[t]$. The following lemma is valid.

Lemma 4.1. An element $G_W{}^\circ[t]$ from $W(t)$, closest in the sense of (4.2), exists for any $t \in [t_0, \vartheta_0]$ and any choice of $G[t]$.

By $S[t]$ we denote the set of all vectors $s[t] = y_W{}^\circ[t] - y[t]$. We define the first player's extremal strategy $U^{(e)} \div u^{(e)}(t, y, G)$ as the function which when $\| y_W{}^\circ[t] - y[t] \| > 0$ associastes with the collection $(t, y[t], G[t])$ any of the vectors $u^{(e)} \in P$ satisfying for at least one $s[t] \in S[t]$ the condition

$$s'[t] f^{(1)}(t, y[t], u^{(e)}) = \max_{u \in P} s'[t] f^{(1)}(t, y[t], u) \tag{4.3}$$

where the prime denotes transposition. When $\| y_W{}^\circ[t] - y[t] \| = 0$, $U^{(e)}$ is identified with any $u \in P$. We say that the signal $z_0^* = z_0^*[t]$ is maximally absorbing for each fixed t if the condition

$$d\,(G\,[t],\ H\,(t,\ z_0^*)) = \min_{z^*} d\,(G\,[t],\ H\,(t,\ z^*))$$

is fulfilled for it, where the minimum is taken over all admissible signals $z^* = z^*\,[t]$ corresponding to points $z \in G\,[t]$. We assume further the fulfillment of the following regularity conditions.

Condition A. A maximally absorbing signal $z_0^*\,[t]$ satisfies the inequality

$$d\,[G\,(t - \Delta_i^{(k)},\ H_W\,(t - \Delta_i^{(k)},\ z^*\,[t - \Delta_i^{(k)}]),\ t),\ \ H_W\,(t,\ z_0^*\,[t])] \leqslant \gamma\,(\Delta_i^{(k)})$$

and $\varepsilon > 0$ can be found such that the sets $H_W{}^{\varepsilon}\,(t, z^*\,[t])$ are canonically closed /10/ for any $t \in \Delta_i^{(k)}$, $\Delta_i^{(k)} = \tau_{i+1}^{(k)} - \tau_i^{(k)}(i,\ k = 0,\ 1, \ldots)$. Here $H_W{}^{\varepsilon}\,(t, z^*\,[t]) = G\,(t - \Delta_i^{(k)}, H_W\,(t - \Delta_i^{(k)},\ z^*\,[t - \Delta_i^{(k)}]), t) \cap S^{\varepsilon}\,(z^*\,[t])$, where $S^{\varepsilon}\,(z^*\,[t])$ is a ring of thickness ε with center at point $z^*\,[t]$ and outer radius β, and $\gamma\,(\Delta_i^{(k)}) \to 0$ as $\Delta_i^{(k)} \to 0$.

Condition B. For any collection $(t,\, y\,[t],\, H\,(t,\, z^*\,[t]))$ for which $d\,(H\,(t, z^*\,[t]),\ G_W{}^{\circ}\,[t]) > 0$ the maximum in

$$d\,(H\,(t,\ z^*\,[t]),\ G_W{}^{\circ}\,[t]) = \max_{z \in H(t,\, z^*[t])} d\,(z,\ G_W{}^{\circ}\,[t]) \tag{4.4}$$

is reached on the unique vector $z_W{}^{\circ}\,[t] - z^{\circ}\,[t]$, where $z_W{}^{\circ}\,[t] \in G_W{}^{\circ\circ}\,[t] \subset G_W{}^{\circ}\,[t]$, $z^{\circ}\,[t] \in H^{\circ}\,[t] \subset H\,(t, z^*\,[t])$, $G_W{}^{\circ\circ}\,[t]$ is the set of elements from $G_W{}^{\circ}\,[t]$ closest to $z^{\circ}\,[t]$, $H^{\circ}\,[t]$ is the set of elements from $H\,(t,\ z^*\,[t])$ on which the maximum in (4.4) is reached.

We note that Conditions A and B are fulfilled, in particular, if the second player's attainability domain is convex. The following lemma is valid.

Lemma 4.2. Let Conditions A and B be fulfilled, let $(y_*, G_*) \in W\,(t_*)\ (t_0 \leqslant t_* \leqslant \vartheta_0)$, and let the system of nonempty closed sets $W\,(t)\ (t_* \leqslant t \leqslant \vartheta_0)$ be u-stable. Then the inclusion $(y\,[t],\ G\,[t]) \in W\,(t)$ is fulfilled up to contact with set M for every motion $y\,[t] = y\,[t,\ t_*,\ y_*,\ G_*,\ U]$ generated by the first player's strategy $U^{(e)}$ extremal to this system of sets $W\,(t)$.

5. We construct the maximal u-stable system of sets $W\,(t, \vartheta_0)$ relative to M. We say that set M is position absorbed from $(t_*,\, y_*,\, G_*)$ by the instant $t = \vartheta_0$ if for any point $z_* \in G_*$, any strategy $V^* \div v\,(t,\, y,\, z)$, and any admissible method of forming the signal $z^*\,[t] = z^*\,(t,\ z\,[t])$ for the motions $z\,[t] = z\,[t,\ t_*,\ z_*,\ V^*]$ we can find a first player's motion $y^*\,[t]$ such that for some information motion $G^*\,(t, z^*\,(t,\, z\,[t]))$ we have

$$G^*\,[t] \subset L\,(y^*\,[t])$$

for some $t \in [t_*,\, \vartheta_0]$. By $W\,(t, \vartheta_0)$ we denote the set of all $(y\,[t],\ G\,[t])$ such that set M is position absorbed from $(t,\ y\,[t],\ G\,[t])$ by the instant ϑ_0.

Lemma 5.1. Each of the sets $W\,(t, \vartheta_0)\ (t_0 \leqslant t \leqslant \vartheta_0)$ is closed in the Hausdorff metric (2.4), the system of sets $W\,(t, \vartheta_0)$ is u-stable relative to M, and $W\,(\vartheta_0, \vartheta_0) = M$.

The validity of the next statement derives from Lemmas 4.2 and 5.1.

Theorem 5.1. The system of sets $W\,(t, \vartheta_0)\ (t_0 \leqslant t \leqslant \vartheta_0)$ of position absorbtion is a maximal u-stable bridge. When Conditions A and B are fulfilled the strategy $U^{(e)} \div u^{(e)}\,(t,\, y,\, G)$ extremal to this bridge solves the problem.

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Translated by N.H.C.